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AN OPTIMUM DEPLOYMENT OF DEFENSIVE WEAPONS

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RDT & E Project Nos. 1M023201A098 & 1M010501A003
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ABSTRACT

Dynamic programming methods are used to determine the allocation of defensive weapons to target areas so as to minimize effects of optimal attack policies of the offense.

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INTRODUCTION

The introduction of long range ballistic missiles, with their large destructive capabilities, in the arsenals of war weapons introduces some new problems in planning the defense of key industrial and military sites. The attack times involved now are such that the defense weapons cannot be redeployed and in fact, no target area can be considered to be inaccessible. Thus, it becomes necessary to determine a defense which will be effective against enemy attacks of unknown duration, strength, location and time. It may happen that through analysis, subject to the hypothesis considered, the supply of defensive weapons is not large enough or the defensive weapon system has excessive target acquisition times to defend the target areas adequately. If so, this indicates areas of weakness that should be remedied either through increased production or improved engineering.

In the following pages we shall describe a method of determining how the defensive weapons should be allocated to the various defense areas. At the same time optimal attack policies will be determined and the best defense against these optimal attacks will be determined. In the assumption involved we shall be sometimes optimistic and sometimes pessimistic about the capabilities of the weapon systems of both sides, the intelligence we have concerning the location of targets, their worth, the number of the defensive weapons and the strength of the attack.

The solution to the problem of the "best" defense will be somewhat of a compromise solution since we will assume that the enemy wishes to do his best and the defense will attempt to do his best against the attack. Thus, the defense chosen will minimize the effect of the attackers best offense against any assumed defense. This defense has the possible drawback effect that if the attacker does not use an optimal attack policy (against some assumed defense) there may exist other defenses which will result in less expected damage. We shall elaborate on this point after formulating and indicating the method of solution of the problem.

Problems involving optimum deployment of attacking missiles against targets which are defended by a known number of defensive missiles have been considered by Perkins[1] and Lindsey[2]. While the problem posed here is somewhat related to the problem considered by Perkins and Lindsey, the computational procedure and formalization of the problem differ. We shall indicate in the following how the problem considered by Perkins and Lindsey may be reformulated and solved using the methods adopted here.

STATEMENT OF THE PROBLEM

Suppose we have n targets which are to be defended by a total number, D, of defensive weapons (e.g., antimissile missiles) against a total number, A, of offensive weapons. We make the following assumptions:

(a) The targets are independent. This implies that once the defensive weapons are allocated to specific targets they cannot be reassigned to other targets during an attack (or an imminent attack) and that attacking weapons that miss their assigned target do not strike another target in the target complex.

(b) The target locations are known to the offense and each side assigns the same worth to a particular target.

(c) The total number of defensive and offensive weapons are known to each side. Their distribution among the targets is not known.

(d) The defensive forces assume that the offense will deploy their weapons in such a way as to maximize the expected damage. That is, if the offense knew of the distribution of the defensive weapons among the target areas then it assigns the attacking missiles in such a way as to maximize the expected damage.

(e) If a missile eludes the defense then the probability of destruction of the target is 1.

(f) The probability that the defensive weapons fail to destroy one or more of the attacking missiles (against the ith target) is a known function, p, of the number of attacking missiles, the number of defensive missiles assigned to the ith target and the probability that a defensive weapon intercepts an attacking missile (if assigned to it). The probability that a defensive weapon intercepts an attacking missile is a constant k, the same for all defensive weapons. Further $p = p(d_i, a_i, k)$ has the properties that $p(0, a_i, k) = 1$ if $a_i > 0$ and $p(0,0,k) = 0$.

(g) Let w_i be worth of target i; then the expected damage to the n targets is $\sum_{i=1}^{n} w_i p(d_i, a_i, k)$.

(h) The defense philosophy is to deploy the defensive weapons in such a fashion so as to minimize the expected damage resulting from the attackers optimal policies.

The problem, then, is to choose a set of d_i's so as to minimize $\underset{\{a_1,\ldots,a_n\}}{\mathrm{Max}} \sum_{i=1}^{n} w_i p(d_i, a_i, k)$, i.e., we wish to determine

$$\underset{\{d_1,\ldots,d_n\}}{\mathrm{Min}} \left[\underset{\{a_1, \ldots, a_n\}}{\mathrm{Max}} \sum_{i=1}^{n} w_i p(d_i, a_i, k) \right] \tag{1}$$

subject to the conditions that $\sum_{i=1}^{n} d_i = D$ and $\sum_{i=1}^{n} a_i = A$.

DYNAMIC PROGRAMMING FORMULATION OF THE PROBLEM

Our goal is to provide a feasible computational procedure to evaluate

$$G(d_1,\ldots,d_n;a_1,\ldots,a_n) = \underset{\{d_1,\ldots,d_n\}}{\mathrm{Min}} \left[\underset{\{a_1,\ldots,a_n\}}{\mathrm{Max}} \sum_{i=1}^{n} w_i p(d_i, a_i, k) \right] \tag{2}$$

subject to the conditions that $A = \sum_{i=1}^{n} a_i$, $D = \sum_{i=1}^{n} d_i$ where A and D are fixed non-negative integers.

To obtain the computational procedure we consider A, D and n to be parameters. In this way we shall be able to reduce this problem to a sequence of simpler problems or stages and at the ith stage we will be able to determine a specific allocation d_i to the ith target. To attain this simplification we use the functional equation technique of dynamic programming and Bellman's Optimality Principle[3]. First we note that the value of G will depend on A, D and n hence, we let

$$f_N(D,A) = \underset{\{d_1,\ldots,d_N\}}{\text{Min}} \left[\underset{\{a_1,\ldots,a_N\}}{\text{Max}} \sum_{i=1}^{N} w_i p(d_i,a_i,k) \right] \text{ for } 1 \leq N \leq n \qquad (3)$$

or

$$f_N(D,A) = \underset{\{d_1,\ldots,d_N\}}{\text{Min}} \left[\underset{\{a_1,\ldots,a_N\}}{\text{Max}} \{w_N p(d_N,a_N,k) + \sum_{i=1}^{N-1} w_i p(d_i,a_i,k) \} \right] \qquad (4)$$

$$1 < N \leq n$$

Now (4) can be written as

$$f_N(D,A) = \underset{d_N}{\text{Min}} \left[\underset{a_N}{\text{Max}} \{w_N p(d_N,a_N,k) + f_{N-1}(D-d_N,A-a_N)\} \right] \qquad (5)$$

To obtain (5) from (4) we have employed Bellman's Optimality Principle[3]. For N = 1 we have

$$f_1(D,A) = w_1 p(D,A,k) \qquad (6)$$

which implies that for a constant A, p decreases as D increases and for constant D, p increases with increasing A. Stated differently it means that if we only have one target to defend (or attack) then we defend (attack) it with our total resources.

Examination of (5), in particular the quantity within the braces shows that as a preliminary calculation we are to calculate optimal attack policies for every defense of the Nth target. This is, in essence, the problem considered by Perkins[2] and Lindsey[2]. For, if we drop the Min operation in equation (2) and assume that D is fixed and each d_i is known the maximum value of $\sum_{i=1}^{n} w_i p(d_i, a_i, k)$ depends only on A and n. Hence defining $g_N(A)$ as the max value of $\sum_{i=1}^{n} w_i p(d_i, a_i, k)$ we have

$$g_N(A) = \max_{a_N} \{ w_N p(a_N, d_N, k) + g_{N-1}(A-a_N) \} \tag{7}$$

and

$$g_1(A) = w_1 p_1(A, d_1, k) \ .$$

The approach, indicated by equation (7), has the advantage of computational simplicity and allows one to assess the effect, on the optimal attack policy, resulting from the addition of more targets and the change in A.

COMPUTATIONAL PROCEDURE

The dynamic programming formulation imbeds the problem within a family of analogous problems in which the parameters, D, A and n assume sets of values. Examination of equations (5) and (6) shows that the first step in the computation is to assign a grid of values for D, A and n. Since D, A and n are non-negative integers then an obvious choice would be to let D, A and n assume the values 0,1,2,... up to the largest values which are of interest. A careful analysis of equations (5) and (6) shows that in the process of calculation we will obtain solutions to a variety of sub problems, that is, problems which may involve smaller numbers of targets, defensive and offensive weapons. Furthermore we use the solutions for these problems to obtain solutions of problems of greater complexity.

The choice or selection of grid values for A and D will, in general, depend on the magnitudes of A and D and also the memory capacity of the available digital computer as well as the time and cost requirements to obtain the solution; hence, no general rules can be formulated to determine what grid size should be selected.

Once the choice of grid values for D and A has been made the calculation starts with equation (6). That is, if δ and α are arbitrary elements of the D and A grid then $f_1(\delta,\alpha)$ is computed, stored and printed for each δ and α

satisfying the relations $0 \leq \delta \leq D$, $0 \leq \alpha < A$. The set of values, $\{f_1(\delta,\alpha)\}$, is then used to compute $f_2(\delta,\alpha)$ from equation (5). For N = 2 equation (5) becomes

$$f_2(\delta,\alpha) = \operatorname*{Min}_{d_2} \{\operatorname*{Max}_{a_2} \left[w_2 p(d_2,a_2,k) + f_1(\delta-d_2,\alpha-a_2) \right] \} .$$

The expression $h(d_2) = \operatorname*{Max}_{a_2} \left[w_2 p(d_2,a_2,k) + f_1(\delta-d_2, \alpha-a_2)\right]$ is a function of d_2 and we must determine the d_2, $0 \leq d_2 \leq \delta$, which minimizes $h(d_2)$. The value d_2 is a function of δ and α and this value gives the allocation of defensive weapons. The allocation of defensive weapons to target 1, is, of course, $d_1 = \delta-d_2$. The set of values $\{f_2(\delta,\alpha) \mid 0 \leq \delta \leq D, 0 \leq \alpha \leq A\}$ is stored over the set of values of $f_1(\delta,\alpha)$ which are no longer needed in the calculation. Also, the sets of values $f_2(\delta,\alpha)$, $d_2(\delta,\alpha)$ and $a_2(\delta,\alpha)$ are printed (and stored if convenient).

The above procedure is repeated until $f_n(\delta,\alpha)$ is obtained. If the only case of interest for the n targets is when $\delta = D$ and $\alpha = A$ then this calculation does not have to be repeated for each grid point.

The structure of the process allows the decision maker to assess the effect on the expected damage due to changes in the number of targets, defensive and offensive weapons, hence the solution may be used as an aid for planning purposes. By this method of calculation we have obtained solutions for 1, 2 or n targets for all ordered pairs, (δ,α) where δ and α are non-negative integers such that $0 \leq \delta \leq D$, $0 \leq \alpha \leq A$. Hence, if at the present time the defense has D_1 defensive weapons, the offense has A_1 offensive weapons whereas in m months the expected numbers of defensive weapons will be D_2, and the expected number of offensive weapons will be A_2 then the solution gives the expected damage for each case.

Further generalization (or restriction) of the problem is possible by modification of the assumptions (a)-(h). For example, assumption (e) is not necessary or realistic for most conventional weapons; that is, it may not be possible to totally destroy an undefended target with the available supply of offensive missiles. In an earlier paper[4], the authors considered optimal attack policies against targets for which the total destruction was not generally possible. Each target had a survival probability associated with it which could be interpreted as a function of its defense and the particular weapon system directed against it. In this earlier paper the resources or supplies of the offense were limited and included several classes of offensive weapon systems.

The problem of decoys has not been mentioned above but may be included by the introduction of proper constraints or by redefining the function $p(d_i, a_i, k)$. If the defensive weapon system has rapid response and short target discrimination times decoys may pose no problem; however since cases for which this is not true exist any realistic problem should include a treatment of decoys. Perkins[1] includes these features in his discussion.

In the following pages we consider a simple numerical example which illustrates the method of solution and information obtained by solving equations (5) and (6) by the above described method. The straight forward solution of equations (5) and (6) as described above requires the storage of a table of $(D+1)(A+1)$ elements in order to proceed to the next stage of the calculations; hence the magnitude of D and A which can be handled is a function of the available high speed memory of the available computer. In our example this poses no problem since we are interested not in solving the most general problem but in illustrating the method of calculation and the kind of information obtained from this method of solution. In our example $p = p(d_i, a_i, k)$ is the function used by Lindsey[2], however, d_i is not known, a priori, but must be determined. As stated previously, this determines the "best" defense against the attackers' optimal attack policies.

AN ILLUSTRATIVE EXAMPLE

$$\text{Let } p = p(d_i, a_i, k) = \begin{cases} 1 - \left[1 - \exp\{-kd_i/a_i\}\right]^{a_i} & \text{if } a_i > 0 \\ 0 & \text{if } a_i = 0 \end{cases}$$

$$A = 10,\ D = 25,\ k = 1/2$$

and suppose we have three targets T_1, T_2, T_3 whose respective worths are $w_1 = 16$, $w_2 = 14$ and $w_3 = 7$. Our goal is to evaluate the function given by equation 2. To accomplish this we imbed the problem into a family of analogous problems, that is, we consider the numbers D and A to be parameters and allow the number of targets to vary also. We let δ and α be any arbitrary non-negative integer less than or equal to D and A, respectively and start the calculation with the computation of $f_1(\delta,\alpha)$ (equation 6) for all admissible values of δ and α. This calculation gives us the expected damage of target T_1 when the defense consists of δ missiles and the attacker has α missiles. The results of these computations are stored in the high speed memory of the computing device and are given also in Table I. Since, for one target, we defend with the total number of defensive weapons available and the attacker attacks with what he has available there is no real need to print the corresponding defense and attack policies. The next step is to compute $f_2(\delta,\alpha)$ for all admissible δ,α from equation (5), with N = 2, i.e.,

$$f_2(\delta,\alpha) = \underset{d_2}{\text{Min}} \left[\underset{a_2}{\text{Max}}\{w_2 p(d_2,a_2,k) + f_1(\delta-d_2, \alpha-a_2)\} \right] . \tag{5}$$

The calculation of $f_2(\delta,\alpha)$ starts with δ = D and α = A and the result $f_2(D,A)$ is stored over $f_1(D,A)$ which is no longer needed in the calculation process. The values $f_2(D,A)$, $d_2(D,A)$ and $a_2(D,A)$ are also printed. Then $f_2(D-1,A)$ is computed and stored over $f_1(D-1,A)$ and $f_2(D-1,A)$, $d_2(D-1,A)$, $a_2(D-1,A)$ are printed. The process is continued until the complete set of values $f_2(\delta,\alpha)$ is computed. At the completion of this stage we have obtained solutions for cases involving 2 targets, δ defensive weapons and α offensive weapons. The expected damage, the allocation of defensive weapons and the optimal attack policy are presented in Tables II, III and IV. Thus, for example, for 2 targets, 20 defensive weapons and 6 attack weapons we find in Table II that the expected damage is 14.66. Similarly we find, in Table III that target T_2 should be defended with 7 defensive weapons, and target T_1 should be defended with 20-7=13 defensive weapons. The optimal attack policy which yields this of expected damage is given in Table IV, i.e., $a_2 = 0$ and $a_1 = 6 - a_2 = 6$. The next stage of the computation is to compute $f_3(\delta,\alpha)$ from equation (5) with N=3, i.e.,

$$f_3(\delta,\alpha) = \underset{d_3}{\mathrm{Min}} \left[\underset{a_3}{\mathrm{Max}} \{w_3 p(d_3,a_3,k) + f_2(\delta-d_3, \alpha-a_3)\} \right] .$$

The results of this calculation are presented in Tables V, VI and VII. At this stage the above example has been solved as well as many other similar problems, viz., solutions to 858 problems have been obtained.

The entry, in Table V, corresponding to D = 25, A = 10 is expected damage on three targets which are defended by 25 defensive weapons against 10 attacking weapons. We find that $f_3(25, 10) = 25.55$. The defense of T_3 and the optimal attack policy against T_3 are given in Tables VI and VII respectively. Thus, we find that $d_3 = 2$ and $a_3 = 2$ which leaves 23 defensive weapons and 8 attack weapons to be allocated to T_1 and T_2. From Table III we find that $d_2 = d_2(23,8) = 9$ and from Table IV $a_2 = a_2(23,8) = 4$ hence $d_1 = 14$ and $a_1 = 4$. Thus, the best defense for the 3 targets (with their 25 defense weapons) against the attackers 10 offensive weapons is $(d_1,d_2,d_3) = (14,9,2)$. This defense of the three targets insures that the expected damage will not exceed 25.55. The attack policy which yields this level of expected damage is $(a_1,a_2,a_3) = (4,4,2)$ and any other attack policy against these three targets and the given defense will result in a lower value of the expected damage. As a further illustration of the above suppose there exist only 20 weapons to defend the three targets and the attacker has six offensive weapons. Then we find that $f_3(20,6) = 17.43$ and the best defense is (10,7,3). The optimal attack policy then is (0,4,2). If the attacker allocates his missiles such that $(a_1,a_2,a_3) = (3,2,1)$ against these three targets with the defense (10,7,3) then the expected damage will be 13.47. Furthermore, if for some reason, the above defense is not acceptable to the defenders, and should be, say (9,8,3) then this results in a penalty for then the maximum expected damage will be greater than 17.43, viz., 17.72 for an attack policy of (4,0,2). This information is not generally printed out but is obtained actually in the course of the calculation. This information may be useful in some instances when we have associated cost functions for then it may be advantageous to have near-optimal policies of defense. The method of solution gives results which may be useful for many purposes, for example, one may construct iso-damage curves from the f_n-tables and thus be able to anticipate future needs of defensive weapons as the attackers supply of offensive missiles increases. Furthermore, from the f_n-tables one can estimate the effect on the expected damage which results from over or under-estimating the number of attack missiles available to the attacker.

It may happen, that because of transportation, cost, defense site planning or structure of the defense system one does not desire the allocation of an individual missile to a target but rather wishes to allocate the defensive weapons in blocks of m. If this is desired then the grid of D- values should be chosen to be 0, m, 2m, ..., D = jm. In fact, such a method actually simplifies the calculation, reduces memory requirements and computation time. Furthermore, the method of solution described above allows one to assess the effect that such an allocation has on the expected damage and yields information as to what the size of the allocation block should be. That is, one actually performs a sensitivity analysis of the problem in the course of solution.

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REFERENCES

1. Perkins, F. M. Optimum Weapon Deployment for Nuclear Attack. Operations Research. Vol. 9, 77-94 (1961).

2. Lindsey, G. R. Optimum Allocation of Offensive and Defensive Forces. Abstracted in: Operations Research/Management Science. Vol. II, No. 4, page 249 (1962).

3. Bellman, R. Dynamic Programming. Princeton University Press (1957).

4. Sacco, W. J. and Shear, R. E. The Optimum Allocation of Weapons to Targets: A Dynamic Programming Approach. Ballistic Research Report No. 1175, Aberdeen Proving Ground, Md. (1962)

TABLE I

$f_1(D,A)$: EXPECTED DAMAGE TO TARGET T_1

D \ A	0	1	2	3	4	5	6	7	8	9	10
0	0.00	16.00	16.00	16.00	16.00	16.00	16.00	16.00	16.00	16.00	16.00
1	0.00	9.70	15.22	15.94	16.00	16.00	16.00	16.00	16.00	16.00	16.00
2	0.00	5.89	13.52	15.64	15.96	16.00	16.00	16.00	16.00	16.00	16.00
3	0.00	3.57	11.54	15.02	15.85	15.98	16.00	16.00	16.00	16.00	16.00
4	0.00	2.16	9.61	14.16	15.62	15.94	15.99	16.00	16.00	16.00	16.00
5	0.00	1.31	7.85	13.11	15.25	15.85	15.97	16.00	16.00	16.00	16.00
6	0.00	0.80	6.34	11.96	14.76	15.70	15.94	15.99	16.00	16.00	16.00
7	0.00	0.48	5.08	10.78	14.15	15.48	15.88	15.98	16.00	16.00	16.00
8	0.00	0.29	4.04	9.61	13.44	15.19	15.79	15.95	15.99	16.00	16.00
9	0.00	0.18	3.20	8.50	12.67	14.82	15.65	15.91	15.98	16.00	16.00
10	0.00	0.11	2.52	7.46	11.85	14.38	15.48	15.86	15.96	15.99	16.00
11	0.00	0.06	1.98	6.51	11.01	13.88	15.25	15.77	15.94	15.99	16.00
12	0.00	0.04	1.55	5.66	10.17	13.33	14.98	15.66	15.90	15.98	15.99
13	0.00	0.02	1.22	4.89	9.34	12.74	14.66	15.52	15.85	15.96	15.99
14	0.00	0.01	0.95	4.22	8.54	12.12	14.29	15.35	15.79	15.94	15.98
15	0.00	0.01	0.74	3.62	7.78	11.47	13.89	15.15	15.70	15.90	15.97
16	0.00	0.00	0.58	3.11	7.06	10.82	13.44	14.91	15.59	15.86	15.96
17	0.00	0.00	0.45	2.66	6.38	10.16	12.98	14.64	15.46	15.81	15.94
18	0.00	0.00	0.35	2.27	5.75	9.52	12.48	14.34	15.31	15.74	15.91
19	0.00	0.00	0.28	1.94	5.17	8.88	11.97	14.01	15.13	15.66	15.88
20	0.00	0.00	0.21	1.65	4.64	8.27	11.44	13.65	14.92	15.56	15.84
21	0.00	0.00	0.17	1.41	4.16	7.67	10.91	13.26	14.70	15.44	15.78
22	0.00	0.00	0.13	1.20	3.72	7.10	10.37	12.86	14.44	15.30	15.72
23	0.00	0.00	0.10	1.02	3.32	6.56	9.84	12.44	14.17	15.15	15.64
24	0.00	0.00	0.08	0.86	2.96	6.05	9.31	12.01	13.87	14.98	15.56
25	0.00	0.00	0.06	0.73	2.63	5.57	8.79	11.57	13.56	14.79	15.45

TABLE II

$f_2(D,A)$: EXPECTED DAMAGE TO TARGET T_2 AND T_2

D \ A	0	1	2	3	4	5	6	7	8	9	10
0	0.00	16.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
1	0.00	14.00	23.70	29.22	29.94	30.00	30.00	30.00	30.00	30.00	30.00
2	0.00	9.70	18.20	23.71	28.53	29.26	29.89	29.95	29.99	30.00	30.00
3	0.00	8.49	14.38	21.54	26.84	28.90	29.58	29.90	29.96	29.99	30.00
4	0.00	5.89	13.31	18.67	24.86	27.47	29.09	29.64	29.92	29.96	29.99
5	0.00	5.15	11.83	16.65	22.92	26.67	28.71	29.50	29.81	29.94	29.98
6	0.00	3.57	11.54	14.76	21.44	25.13	27.84	28.99	29.62	29.85	29.95
7	0.00	3.12	10.10	13.68	19.69	23.93	27.09	28.69	29.48	29.79	29.92
8	0.00	2.16	9.61	13.15	17.96	22.56	26.26	28.00	29.12	29.60	29.84
9	0.00	1.89	8.40	13.11	16.26	21.08	25.10	27.50	28.76	29.48	29.79
10	0.00	1.31	7.85	12.39	14.76	19.98	24.25	26.72	28.42	29.20	29.65
11	0.00	1.15	6.87	11.96	14.14	18.66	23.16	26.02	27.87	28.99	29.51
12	0.00	0.80	6.34	11.47	13.66	17.51	22.00	25.22	27.45	28.62	29.35
13	0.00	0.70	5.55	10.78	13.44	16.33	21.08	24.34	26.79	28.31	29.13
14	0.00	0.48	5.08	10.46	13.35	15.19	19.97	23.58	26.29	27.86	28.93
15	0.00	0.42	4.44	9.61	12.91	14.38	18.93	22.56	25.59	27.44	28.57
16	0.00	0.29	4.04	9.43	12.67	13.88	17.92	21.85	24.85	26.95	28.30
17	0.00	0.26	3.53	8.49	12.38	13.87	16.89	20.88	24.23	26.42	27.93
18	0.00	0.18	3.20	8.41	11.85	13.74	15.87	19.98	23.39	25.91	27.63
19	0.00	0.16	2.80	7.46	11.76	13.55	15.09	19.20	22.76	25.26	27.18
20	0.00	0.11	2.52	7.44	11.09	13.33	14.66	18.37	21.94	24.76	26.75
21	0.00	0.09	2.20	6.53	11.01	13.29	14.09	17.54	21.15	24.02	26.30
22	0.00	0.07	1.98	6.51	10.37	12.97	13.90	16.70	20.43	23.52	25.75
23	0.00	0.06	1.73	5.70	10.17	12.74	13.89	15.87	19.63	22.79	25.33
24	0.00	0.04	1.55	5.66	9.63	12.59	13.81	15.21	18.91	22.16	24.70
25	0.00	0.03	1.36	4.95	9.34	12.15	13.70	14.79	18.15	21.51	24.26

TABLE III

$d_2(D,A)$: ALLOCATION OF DEFENSIVE WEAPONS TO TARGET T_2

D \ A	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	1	0	0
2	0	1	1	1	1	1	1	1	1	1	1
3	0	1	1	2	1	2	1	2	1	1	1
4	0	2	1	2	1	2	3	2	3	2	2
5	0	2	2	3	1	3	2	3	2	1	2
6	0	3	3	2	2	3	2	3	2	3	4
7	0	3	3	2	2	4	2	4	3	4	3
8	0	4	4	3	3	4	3	4	3	4	3
9	0	4	4	4	4	5	3	5	6	5	4
10	0	5	5	4	4	5	3	5	4	5	4
11	0	5	5	5	3	6	4	6	4	6	7
12	0	6	6	5	4	6	4	6	8	6	5
13	0	6	6	6	5	6	4	7	5	7	5
14	0	7	7	6	5	6	5	7	5	7	6
15	0	7	7	7	6	5	5	8	6	8	6
16	0	8	8	7	7	5	6	8	6	8	10
17	0	8	8	8	7	5	7	9	7	9	7
18	0	9	9	8	8	6	8	10	7	9	11
19	0	9	9	9	8	7	7	10	7	10	8
20	0	10	10	9	9	8	7	11	8	10	8
21	0	10	10	10	10	8	6	10	8	11	9
22	0	11	11	11	10	9	7	10	9	12	9
23	0	11	11	11	11	10	8	11	9	12	10
24	0	12	12	12	11	10	8	9	9	13	10
25	0	12	12	12	12	11	9	8	10	13	10

TABLE IV

$a_2(D,A)$: ALLOCATION OF OFFENSIVE WEAPONS TO TARGET T_2

D \ A	0	1	2	3	4	5	6	7	8	9	10
0	0	0	1	2	3	4	5	6	7	8	9
1	0	1	1	1	1	1	1	1	7	2	3
2	0	0	1	1	2	2	3	3	4	4	5
3	0	1	1	2	2	3	3	4	4	4	4
4	0	0	2	1	2	2	3	3	5	4	5
5	0	1	2	1	2	3	3	4	4	3	5
6	0	0	0	1	2	2	3	3	3	4	6
7	0	1	2	3	2	3	2	4	4	5	5
8	0	0	0	3	2	2	3	3	4	4	4
9	0	1	2	0	2	3	3	4	5	5	5
10	0	0	0	3	0	2	2	3	4	4	5
11	0	1	2	0	2	2	3	4	3	5	6
12	0	0	0	3	4	2	3	3	5	4	5
13	0	1	2	0	0	2	2	4	4	5	4
14	0	0	0	3	4	0	3	3	3	4	5
15	0	1	2	0	4	0	3	3	4	5	4
16	0	0	0	3	0	0	3	3	3	4	6
17	0	1	2	0	4	5	3	3	4	5	5
18	0	0	0	3	0	5	3	4	4	4	5
19	0	1	2	0	4	5	3	3	3	5	5
20	0	0	0	3	4	0	0	3	4	4	4
21	0	1	2	3	0	5	3	3	3	4	5
22	0	0	0	0	4	5	6	3	4	5	4
23	0	1	2	3	0	0	0	3	4	4	5
24	0	0	0	0	4	5	6	3	3	5	5
25	0	1	2	3	0	5	6	3	4	4	4

TABLE V

$f_3(D,A)$: EXPECTED DAMAGE TO TARGETS T_1, T_2 AND T_3

D \ A	0	1	2	3	4	5	6	7	8	9	10
0	0.00	16.00	30.00	37.00	37.00	37.00	37.00	37.00	37.00	37.00	37.00
1	0.00	14.00	23.70	30.70	36.22	36.94	37.00	37.00	37.00	37.00	37.00
2	0.00	9.70	18.20	25.20	30.71	35.53	36.26	36.89	36.95	36.99	37.00
3	0.00	8.49	15.49	21.54	28.53	32.78	35.19	35.91	36.55	36.87	36.92
4	0.00	7.00	13.31	20.31	25.67	31.08	33.50	35.37	36.10	36.57	36.87
5	0.00	5.89	12.15	18.67	23.65	29.11	31.71	34.13	35.74	36.30	36.62
6	0.00	5.15	11.55	16.65	21.76	27.17	30.78	33.33	35.01	35.93	36.48
7	0.00	4.25	10.12	15.79	20.68	25.69	29.25	30.08	34.14	35.55	36.22
8	0.00	3.57	9.61	14.35	19.69	23.93	27.84	31.04	33.66	34.91	35.83
9	0.00	3.12	8.89	13.68	17.96	22.56	26.81	29.85	32.89	34.41	35.53
10	0.00	2.58	8.31	13.11	17.35	21.08	25.32	28.83	31.76	33.74	35.04
11	0.00	2.16	7.85	12.39	16.26	19.98	24.25	27.68	30.97	33.05	34.58
12	0.00	1.89	6.87	11.96	14.96	18.66	23.16	26.83	30.16	32.45	34.08
13	0.00	1.56	6.66	11.47	14.53	18.13	22.00	26.02	29.08	31.71	33.48
14	0.00	1.31	6.34	10.78	14.04	17.51	21.08	25.22	27.91	30.93	32.92
15	0.00	1.15	5.92	10.46	13.45	16.69	20.00	24.21	26.99	30.16	32.22
16	0.00	0.95	5.55	9.61	13.35	16.33	19.57	23.18	26.29	29.39	31.65
17	0.00	0.80	5.08	9.43	12.91	15.52	18.93	22.17	25.59	28.48	30.96
18	0.00	0.70	5.05	8.50	12.67	14.66	18.59	21.14	24.85	27.61	30.23
19	0.00	0.57	4.44	8.41	12.38	14.39	17.93	20.40	24.23	26.81	29.57
20	0.00	0.48	4.20	7.46	11.76	13.89	17.43	19.98	23.39	25.97	29.02
21	0.00	0.42	4.04	7.44	11.68	13.74	16.89	19.46	22.62	25.34	28.30
22	0.00	0.35	3.53	6.97	11.01	13.55	16.06	19.20	21.79	24.76	27.63
23	0.00	0.29	3.44	6.84	10.76	13.33	15.87	18.38	20.94	24.02	26.99
24	0.00	0.26	3.20	6.57	10.17	12.97	15.20	18.34	20.43	23.52	26.30
25	0.00	0.21	2.80	6.53	9.90	12.75	14.66	17.54	19.81	22.79	25.55

TABLE VI

$d_3(D,A)$: ALLOCATION OF DEFENSIVE WEAPONS TO TARGET T_3

D \ A	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	1	1	1	1	1	1	1
4	0	0	0	0	0	1	1	2	2	1	2
5	0	1	0	1	0	1	1	1	2	1	1
6	0	1	0	1	0	1	2	1	2	2	1
7	0	1	0	1	0	1	2	1	3	2	3
8	0	2	0	1	1	1	2	2	4	2	2
9	0	2	0	2	1	1	1	2	3	3	2
10	0	2	0	1	1	1	1	2	1	3	2
11	0	3	1	1	2	1	1	2	1	3	3
12	0	3	1	1	2	1	1	2	3	4	3
13	0	3	1	1	2	1	1	2	2	4	3
14	0	4	2	1	2	2	1	2	2	4	4
15	0	4	2	1	2	2	1	1	2	5	4
16	0	4	3	1	2	3	1	1	2	3	4
17	0	5	3	1	2	3	2	1	2	2	5
18	0	5	3	1	2	3	2	1	2	3	5
19	0	5	4	1	2	4	3	1	1	2	6
20	0	6	4	1	1	4	3	2	2	2	5
21	0	6	5	1	1	3	4	2	1	2	5
22	0	6	5	1	1	3	4	3	1	2	4
23	0	7	5	2	1	3	5	3	1	2	3
24	0	7	6	3	1	2	5	3	2	2	3
25	0	7	6	4	1	2	5	4	2	2	2

TABLE VII

$a_3(D,A)$: ALLOCATION OF OFFENSIVE WEAPONS TO TARGET T_3

D \ A	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	1	2	3	4	5	6	7	8
1	0	0	0	1	1	1	1	1	1	1	1
2	0	0	0	1	1	1	1	1	1	1	1
3	0	0	1	0	0	1	2	2	2	3	3
4	0	1	0	1	1	1	2	3	3	2	4
5	0	0	1	0	1	1	1	2	3	2	3
6	0	0	0	0	1	1	2	2	2	3	3
7	0	1	1	1	1	1	1	1	2	3	3
8	0	0	0	1	0	1	0	2	3	2	3
9	0	0	1	0	0	0	1	2	2	3	3
10	0	1	1	0	1	0	1	1	2	2	2
11	0	0	0	0	0	0	0	1	1	2	3
12	0	0	0	0	1	0	0	1	2	3	3
13	0	1	2	0	1	2	0	0	2	2	2
14	0	0	0	0	1	0	0	0	2	2	3
15	0	0	2	0	0	2	2	1	2	2	3
16	0	1	0	0	0	0	2	1	0	2	2
17	0	0	0	0	0	2	0	1	0	2	3
18	0	0	2	0	0	2	2	1	0	2	2
19	0	1	0	0	0	0	0	2	1	1	3
20	0	0	2	0	0	0	2	0	0	1	2
21	0	0	0	0	1	0	0	2	1	1	0
22	0	1	0	3	0	0	2	0	1	0	0
23	0	0	2	3	1	0	0	2	1	0	2
24	0	0	0	3	0	0	2	2	0	0	0
25	0	1	0	0	1	1	0	0	2	0	2

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